# Calculating the Magnetoresistance Effect in Metals

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Z. Naturforsch. **66a,** 311 – 320 (2011); received September 15, 2010

Transversal magnetoresistance is calculated for numerous metal cases on the basis of simple electron theory. Any metal can be represented by a single band of states having a closed Fermi surface which is assumed to be similar in shape to a sphere. In an external electromagnetic field the electron transport seems to be regulated by two kinds of relaxation times. The first one is due to the electric field, and its size is not appreciably influenced by that field. On the other hand, electron motion in the magnetic field is associated with a relaxation time that is strongly dependent on the strength of that field. Both time parameters combine to an effective relaxation time according to Matthiessen's rule. A good agreement between experiment and theory is obtained for Li, Cu, Ag, Au and Pd, Pt metals.

*Key words:* Metals; Magnetoresistance Effect; Relaxation Time.

## 1. Introduction

The well-known experimental problem of magnetoresistance in metals seems to have never been approached using a satisfactory computational theory. The main data involved in calculations, and affecting the results, should be the intensity of the magnetic field and the band-structure parameters characterizing the metal. Nevertheless, the temperature at which the experiments were performed also plays an important role. The aim of the present paper is to suggest a simple theory of magnetoresistance in which all the phenomena mentioned above are taken into account simultaneously. The properties of the band-structure effect can be simplified with the use of a semiclassical approach. Here, in the first step, the electron orbits induced by the magnetic field on the Fermi surface of a metal should be noted [1, 2]. For the sake of simplicity only a single band of the electron states can be assumed. The Fermi surface of such a metal is considered to be almost a closed entity that is similar in shape to a sphere. It will be demonstrated that the description of electron magnetotransport in this metal model is not a difficult task. It should be noted that a former theory of magnetoresistance did not give any numerical insight into the resistance properties of individual metals. Only a general form of dependence of the magnetoresistance on the magnetic induction has been examined with no calculation of the temperature dependence of the metal data [3 – 12].

## 2. The Basic Formulae

When there is no magnetic field (the magnetic induction $B = 0$) acting on a metal, which is assumed to be isotropic with respect to the Cartesian coordinate system, the electric resistance $\rho$ is represented by a tensor

$$\bar{\bar{\rho}}(0) = \frac{m}{n_s e^2 \tau(0)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where $m$ is the electron mass, $n_s$ is the concentration of electron carriers, and $\tau(0)$ the relaxation time due to the response of the metal system to the electric field alone. $\tau(0)$ is a heavily temperature-dependent parameter.

In the next step, when the magnetic field ($B \neq 0$) is acting in parallel to the $z$-axis on a metal having the resistance (1), the resistance tensor becomes [13]

$$\bar{\bar{\rho}}(B) = \frac{m}{n_s e^2 \tau(B)} \begin{pmatrix} 1 & -\xi & 0 \\ \xi & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

where

$$\xi = \tau(B)\Omega_0. \tag{2a}$$

Here, $\tau(B)$ is the relaxation time in the presence of a non-zero magnetic field induction $B$, and

$$\Omega_0 = \frac{eB}{mc} \tag{3}$$

is the frequency of the electron gyration.

It is important to note that when the tensor (2) acts on some column vector, the presence of $\tau(B)$ coming from the non-diagonal terms in (2) [see (2a)] is cancelled because of $\tau(B)$ entering the denominator in (2). This property does not apply to the diagonal terms of (2). The reciprocal values of $\tau(B)$ coming from these terms remain present in the transformed vector.

In numerous cases, especially those concerning semiconductors, the mass $m$ entering (3) is considered to be very different from that of free electrons, but in many metals these differences are less dramatic.

Our attention is now focused on the relaxation time parameter $\tau(B)$. Usually it is assumed, less or more tacitly, that $\tau(B)$ is relatively similar to $\tau(0)$. However, this point of view will be reconsidered in the present paper. In fact, the properties of $\tau(B)$ should be considered quite different from those of $\tau(0)$.

Experimentally, in examining the magnetoresistance, the change $\Delta\rho$ of the electric resistance due to the magnetic field $B$ is measured first, and is next referred to the original resistance $\rho(0)$. Our aim is to show that only a cooperation of both relaxation times, $\tau(0)$ and $\tau(B)$, can lead to a proper estimate of the ratio

$$\frac{\Delta\rho}{\rho(0)} \tag{4}$$

characteristic for the magnetoresistance effect.

## 3. Electron Scattering in the Magnetic Field

Beginning with the action of the electric field alone, the electron motion accelerated by that field is changed because some defects meet electrons and force them to change their momentum. Many of these scattering processes change the momentum direction exactly, or almost exactly, into the opposite of the original momentum direction. However, the same kind of the momentum change is obtained precisely because of the action of the magnetic field alone. For, in the course of electron circulation in the magnetic field, the journey of an electron in some direction that began at a certain orbit point is changed into a journey in the opposite direction from the opposite point of the same planar orbit. This process is repeated periodically with a period regulated by a (constant) frequency $\Omega_0$.

Let the orbit approximate a circle having the radius $R$. Then, a change of direction into the opposite direction is obtained in any orbit when half of the electron circulation is performed. The length $l$ of a free path in a chosen direction is approximately

$$l = 2R. \tag{5}$$

This path is covered in course of the time interval $T_{\mathrm{per}}/2$, where $T_{\mathrm{per}}$ is a full period of the electron circulation.

Evidently, because of (3), the period $T_{\mathrm{per}}$ depends on $B$:

$$T_{\mathrm{per}} = \frac{2\pi}{\Omega_0} = \frac{2\pi mc}{eB}. \tag{6}$$

The relaxation time $\tau$ is, in general, defined as the time necessary for an electron to travel a free path of length $l$. In the present case, this is

$$\tau = \tau(B) = \frac{T_{\mathrm{per}}}{2} = \frac{\pi}{\Omega_0}. \tag{7}$$

Therefore, in view of (6), the relaxation time $\tau(B)$ is inversely proportional to $B$. On the other hand, because of (7), the product $\tau\Omega_0$ is a constant number that approaches

$$\xi = \tau(B)\Omega_0 = \pi. \tag{8}$$

In the next section (Section 4), the constant property of $\xi$ is confirmed by a direct quantum-mechanical calculation.

## 4. $\xi$ Calculated from Equations for the Electron Magnetotransport

The transport properties of an individual electron are described by the motion of an electron in a viscous medium with the friction coefficient $\chi$. The motion is performed upon the Lorentz force due to the action of a coupled electric and magnetic field. The parameter $\chi$ is next considered as a reciprocal value of the relaxation time $\tau$ of the charge carriers (see e. g. [14]):

$$\chi = \frac{1}{\tau}. \tag{9}$$

If a contribution of the electric field $\vec{E}$ is neglected, and only the magnetic field $\vec{B}$ remains of importance [15], so $\tau = \tau(B)$, the equation for the velocity vector $\vec{v}$ becomes [13, 16]:

$$m\left(\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} + \frac{\vec{v}}{\tau}\right) = \frac{e}{c}(\vec{v} \times \vec{B}). \tag{10}$$

Equation (10) is examined below when the quantum properties of $\vec{v}$ and $\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$ are taken into account. This means that dynamical variables of (10) are transformed into a quantum-mechanical operator form. A classical free-electron Hamiltonian is [17]

$$H = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2) \tag{11}$$

in which, for the magnetic field taken along the $z$-axis, we have

$$v_x = \frac{1}{m}\left(p_x - \frac{eB}{2c}y\right), \tag{12}$$

$$v_y = \frac{1}{m}\left(p_y + \frac{eB}{2c}x\right), \tag{13}$$

$$v_z = \frac{1}{m}p_z. \tag{14}$$

Simultaneously, because of

$$B_x = B_y = 0, \tag{15}$$

equation (10) can be extended to the equation pair:

$$m\left(\frac{\mathrm{d}v_x}{\mathrm{d}t} + \frac{v_x}{\tau}\right) = \frac{e}{c}v_y B, \tag{16}$$

$$m\left(\frac{\mathrm{d}v_y}{\mathrm{d}t} + \frac{v_y}{\tau}\right) = -\frac{e}{c}v_x B. \tag{17}$$

The quantum operators of velocity are

$$\hat{v}_x \equiv \frac{\mathrm{d}\hat{x}}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}(x\hat{H} - \hat{H}x) = \frac{1}{m}\left(\hat{p}_x - \frac{eB}{2c}y\right), \tag{12a}$$

$$\hat{v}_y \equiv \frac{\mathrm{d}\hat{y}}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}(y\hat{H} - \hat{H}y) = \frac{1}{m}\left(\hat{p}_y + \frac{eB}{2c}x\right), \tag{12b}$$

$$\hat{v}_z \equiv \frac{\mathrm{d}\hat{z}}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}(z\hat{H} - \hat{H}z) = \frac{1}{m}\hat{p}_z, \tag{12c}$$

where according to (11)

$$\hat{H} = \frac{1}{2}m(\hat{v}_x^2 + \hat{v}_y^2 + \hat{v}_z^2). \tag{18}$$

The quantum operators of acceleration, which are

$$\frac{\mathrm{d}\hat{v}_x}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}(\hat{v}_x\hat{H} - \hat{H}\hat{v}_x), \tag{19}$$

$$\frac{\mathrm{d}\hat{v}_y}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}(\hat{v}_y\hat{H} - \hat{H}\hat{v}_y), \tag{20}$$

can be calculated on the basis of the following commutators [18]:

$$\begin{aligned}\hat{p}_x\hat{H} - \hat{H}\hat{p}_x &= \frac{eB}{2mc}\hat{p}_y(\hat{p}_x x - x\hat{p}_x) \\ &\quad + \frac{1}{2m}\left(\frac{eB}{2c}\right)^2(\hat{p}_x x^2 - x^2\hat{p}_x) \\ &= -\frac{eB}{2mc}\hat{p}_y\mathrm{i}\hbar + \frac{1}{2m}\left(\frac{eB}{2c}\right)^2(-2\mathrm{i}\hbar x),\end{aligned} \tag{21a}$$

$$\begin{aligned}y\hat{H} - \hat{H}y &= \frac{1}{2m}\left[y\hat{p}_y^2 - \hat{p}_y^2 y + \frac{eB}{c}x(y\hat{p}_y - \hat{p}_y y)\right] \\ &= \mathrm{i}\hbar\frac{\hat{p}_y}{m} + \frac{1}{2m}\frac{eB}{c}\mathrm{i}\hbar x,\end{aligned} \tag{21b}$$

applied in the case of operator (19) due to (12a), whereas

$$\begin{aligned}\hat{p}_y\hat{H} - \hat{H}\hat{p}_y &= -\frac{eB}{2mc}\hat{p}_x(\hat{p}_y y - y\hat{p}_y) \\ &\quad + \frac{1}{2m}\left(\frac{eB}{2c}\right)^2(\hat{p}_y y^2 - y^2\hat{p}_y) \\ &= \frac{eB}{2mc}\hat{p}_x\mathrm{i}\hbar + \frac{1}{2m}\left(\frac{eB}{2c}\right)^2(-2\mathrm{i}\hbar y),\end{aligned} \tag{22a}$$

$$\begin{aligned}x\hat{H} - \hat{H}x &= \frac{1}{2m}\left[-\frac{eB}{c}y(x\hat{p}_x - \hat{p}_x x) + x\hat{p}_x^2 - \hat{p}_x^2 x\right] \\ &= \mathrm{i}\hbar\frac{\hat{p}_x}{m} - \frac{1}{2m}\frac{eB}{c}\mathrm{i}\hbar y,\end{aligned} \tag{22b}$$

are applied in the case of operator (20) due to (12b). Evidently, the term

$$\frac{1}{2}m\hat{v}_z^2 = \frac{1}{2m}\hat{p}_z^2 \tag{23}$$

present in $\hat{H}$ does not provide any contribution to the commutators.

Because of (12a) and from (21a) and (21b), the following result is obtained for commutator (19):

$$\begin{aligned}\frac{\mathrm{d}\hat{v}_x}{\mathrm{d}t} &= \frac{1}{\mathrm{i}\hbar m}\left[(\hat{p}_x\hat{H} - \hat{H}\hat{p}_x) - \frac{eB}{2c}(y\hat{H} - \hat{H}y)\right] \\ &= \frac{1}{\mathrm{i}\hbar m}\left[-\frac{eB}{2mc}\hat{p}_y\mathrm{i}\hbar - \frac{1}{m}\left(\frac{eB}{2c}\right)^2\mathrm{i}\hbar x \right. \\ &\qquad \left. - \frac{eB}{2c}\left(\mathrm{i}\hbar\frac{\hat{p}_y}{m} + \frac{eB}{2mc}\mathrm{i}\hbar x\right)\right] \\ &= \frac{1}{m}\left[-\frac{eB}{mc}\hat{p}_y - \frac{2}{m}\left(\frac{eB}{2c}\right)^2 x\right] \\ &= -\frac{eB}{cm^2}\left(\hat{p}_y + \frac{eB}{2c}x\right).\end{aligned} \tag{24}$$

For the next step, because of (13a), the results of (22a) and (22b) produce the commutator in (20)

$$\begin{aligned}\frac{\mathrm{d}\hat{v}_y}{\mathrm{d}t} &= \frac{1}{\mathrm{i}\hbar m}\left[(\hat{p}_y\hat{H}-\hat{H}\hat{p}_y)+\frac{eB}{2c}(x\hat{H}-\hat{H}x)\right]\\ &= \frac{1}{\mathrm{i}\hbar m}\left[\mathrm{i}\hbar\hat{p}_x\frac{eB}{2mc}-\frac{1}{m}\left(\frac{eB}{2c}\right)^2\mathrm{i}\hbar y\right.\\ &\quad\left.+\frac{eB}{2c}\left(\mathrm{i}\hbar\frac{\hat{p}_x}{m}-\frac{eB}{2mc}\mathrm{i}\hbar y\right)\right] \qquad (25)\\ &= \frac{1}{m}\left[\frac{eB}{mc}\hat{p}_x-\frac{2}{m}\left(\frac{eB}{2c}\right)^2 y\right]\\ &= \frac{eB}{cm^2}\left(\hat{p}_x-\frac{eB}{2c}y\right).\end{aligned}$$

The next transformation can be carried out by considering (3). Based on (12b) and (3), the result in (24) is simplified to

$$\frac{d\hat{v}_x}{\mathrm{d}t} = -\Omega_0\hat{v}_y, \qquad (26)$$

whereas, based on (12a) and (3), the result in (25) is simplified to:

$$\frac{d\hat{v}_y}{\mathrm{d}t} = \Omega_0\hat{v}_x. \qquad (27)$$

Expressions (26) and (27) can be used next in the quantized formulae for the transport equations (16) and (17) which become

$$m\left(\frac{\mathrm{d}\hat{v}_x}{\mathrm{d}t}+\frac{\hat{v}_x}{\tau}\right) = m\left(-\Omega_0\hat{v}_y+\frac{\hat{v}_x}{\tau}\right) = \frac{e}{c}\hat{v}_y B, \quad (28)$$

$$m\left(\frac{\mathrm{d}\hat{v}_y}{\mathrm{d}t}+\frac{\hat{v}_y}{\tau}\right) = m\left(\Omega_0\hat{v}_x+\frac{\hat{v}_y}{\tau}\right) = -\frac{e}{c}\hat{v}_x B. \quad (29)$$

By taking into account (3) and (26), we have, from (28), the equation

$$-\Omega_0\hat{v}_y+\frac{\hat{v}_x}{\tau} = \frac{eB}{mc}\hat{v}_y = \Omega_0\hat{v}_y \qquad (30)$$

or

$$\frac{\hat{v}_x}{\tau} = 2\Omega_0\hat{v}_y. \qquad (30a)$$

In the same way, (29) can be transformed into

$$\Omega_0\hat{v}_x+\frac{\hat{v}_y}{\tau} = -\frac{eB}{mc}\hat{v}_x = -\Omega_0\hat{v}_x \qquad (31)$$

$$\frac{\hat{v}_y}{\tau} = -2\Omega_0\hat{v}_x. \qquad (31a)$$

The squared values of (30a) and (31a) can be calculated and added together. These give the equation

$$\frac{1}{\tau^2}(\hat{v}_x^2+\hat{v}_y^2) = 4\Omega_0^2(\hat{v}_x^2+\hat{v}_y^2) \qquad (32)$$

from which we obtain

$$\frac{1}{\tau^2\Omega_0^2} = 4. \qquad (33)$$

Therefore, the product

$$\xi = \tau\Omega_0 = \frac{1}{2} \qquad (34)$$

is a definite quantity independent of the strength of the field $B$ entering the expression for the frequency $\Omega_0$ in (3). Similar $\xi$ can be obtained for the electron transport considered quantum-mechanically in crystal lattices having high point-group symmetry. The physical consequences of this fact are discussed below.

## 5. The Tensor of Magnetoresistance Modified by the Effect of a Constant $\xi$

In numerous experiments, the attention is focused on a transversal magnetoresistance which means that the current is measured in a plane that is normal to the magnetic field. In the present paper, explicit calculations are limited to this case. In view of Sections 3 and 4, two relaxation times should be distinguished. One, labeled by $\tau(0)$, exists in the absence of the magnetic field ($B = 0$) and regulates the electron transport that is due to the electric field alone. This $\tau(0)$ is completely different from another relaxation time, labeled $\tau(B)$, which regulates the motion in the magnetic field and enters a part of the magnetoresistance tensor, depending effectively on $B$. Both relaxation times are assumed to satisfy the Matthiessen rule. Therefore, they add up to an effective relaxation time according to the formula

$$\frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau(0)}+\frac{1}{\tau(B)}. \qquad (35)$$

A constant $\xi$, for example that of (8) or (34), implies the relation

$$\frac{1}{\tau(B)} = \frac{\Omega_0}{\xi}. \qquad (36)$$

Assuming for $\bar{\bar{\rho}}(0)$ the isotropic tensor of (1) and for $\bar{\bar{\rho}}(B_z)$ the magnetoresistance tensor represented in (2),

Table 1. Transversal magnetoresistance of Ia group (alkali) metals calculated from the formula (39) compared with the experimental data [19]. The empirical correcting factor $r$ [19] fits $\tau(0)$ taken from Table 10 to the values corresponding with the temperature of the measurements.

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Li, polycrystal: | | | | |
| 78 | 0.137 | 100 | 0.036 | 0.02 |
| 78 | 0.137 | 200 | 0.072 | 0.08 |
| 78 | 0.137 | 300 | 0.108 | 0.15 |
| 20.4 | 0.0243 | 30.4 | 0.062 | 0.11 |
| Na, polycrystal: | | | | |
| 80 | 0.2 | 300 | 0.269 | 0.07 |
| Na, single crystal: | | | | |
| 20.4 | 0.0043 | 16 | 0.667 | 0.08 |
| 20.4 | 0.0043 | 25 | 1.042 | 0.11 |
| 20.4 | 0.0043 | 31.1 | 1.297 | 0.15 |
| 20.4 | 0.0043 | 39.5 | 1.647 | 0.19 |
| Na, polycrystal: | | | | |
| 20.4 | 0.00675 | 9.36 | 0.249 | 0.07 |
| 20.4 | 0.00675 | 15.6 | 0.414 | 0.15 |
| 20.4 | 0.00675 | 22 | 0.584 | 0.23 |
| 20.4 | 0.00675 | 31.5 | 0.837 | 0.41 |
| 20.4 | 0.00675 | 35.1 | 0.932 | 0.50 |
| K, polycrystal: | | | | |
| 20.4 | 0.0247 | 15.45 | 0.144 | 0.018 |
| 20.4 | 0.0247 | 22.2 | 0.206 | 0.029 |
| 20.4 | 0.0247 | 31.3 | 0.291 | 0.43 |
| 20.4 | 0.0247 | 35.1 | 0.326 | 0.50 |
| Rb, polycrystal: | | | | |
| 14 | 0.0339 | 40 | 0.185 | 0.004 |
| Cs, polycrystal: | | | | |
| 20.4 | 0.0746 | 40 | 0.063 | 0.03 |

we obtain for a full tensor of magnetoresistance in the field $B = B_z$ the expression

$$\bar{\bar{\rho}}^{\rm tot}(B) = \bar{\bar{\rho}}(0) + \bar{\bar{\rho}}(B) = \frac{m}{n_s e^2\tau(0)}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} + \frac{m}{n_s e^2\tau(B)}\begin{pmatrix}1 & -\xi & 0\\ \xi & 1 & 0\\ 0 & 0 & 1\end{pmatrix}. \quad (37)$$

Because a constant $\xi$ is established for the electron transport in the $(x,y)$-plane, the practical applications of formula (37) to the transversal magnetoresistance can be limited only to that plane. In this case, (37) is reduced to

$$\begin{aligned}\bar{\bar{\rho}}^{\rm tot}(B) &= \frac{m}{n_s e^2\tau(0)}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{m}{n_s e^2\tau(B)}\begin{pmatrix}1 & -\xi\\ \xi & 1\end{pmatrix}\\ &= \frac{m}{n_s e^2\tau(0)}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{B_z}{n_s e\xi c}\begin{pmatrix}1 & -\xi\\ \xi & 1\end{pmatrix}.\end{aligned} \quad (38)$$

Table 2. Transversal magnetoresistance of IIa group metals calculated from the formula (39) compared with the experimental data [19]. The empirical correcting factor $r$ [19] fits $\tau(0)$ taken from Table 10 to the values corresponding with the temperature of the measurements.

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Be, polycrystal: | | | | |
| 291 | 1 | 100 | 0.003 | 0.1 |
| 291 | 1 | 200 | 0.006 | 0.35 |
| 291 | 1 | 300 | 0.009 | 0.66 |
| 78 | 0.33 | 100 | 0.009 | 0.4 |
| 78 | 0.33 | 200 | 0.017 | 1.3 |
| 78 | 0.33 | 300 | 0.026 | 2.3 |
| Mg, polycrystal: | | | | |
| 291 | 1 | 100 | 0.006 | 0.02 |
| 291 | 1 | 200 | 0.012 | 0.08 |
| 291 | 1 | 300 | 0.018 | 0.17 |
| 195 | 0.68 | 100 | 0.009 | 0.05 |
| 195 | 0.68 | 200 | 0.018 | 0.17 |
| 195 | 0.68 | 300 | 0.027 | 0.3 |
| 4.21 | 0.00516 | 5 | 0.06 | 1.9 |
| 4.21 | 0.00516 | 10 | 0.119 | 4.7 |
| 4.21 | 0.00516 | 15 | 0.179 | 8.0 |
| 4.21 | 0.00516 | 20 | 0.239 | 11.9 |
| Ba, polycrystal: | | | | |
| 20.4 | 0.0275 | 8.5 | 0.003 | 0.04 |
| 20.4 | 0.0275 | 18 | 0.007 | 0.14 |
| 20.4 | 0.0275 | 26.9 | 0.01 | 0.28 |
| 20.4 | 0.0275 | 33.2 | 0.013 | 0.41 |
| 14 | 0.0152 | 17.63 | 0.012 | 0.34 |
| 14 | 0.0152 | 23.3 | 0.016 | 0.54 |
| 14 | 0.0152 | 26.6 | 0.019 | 0.66 |
| 14 | 0.0152 | 33.2 | 0.023 | 0.96 |
| Ba, polycrystal: | | | | |
| 4.22 | 0.0079 | 4.5 | 0.006 | 0.11 |
| 4.22 | 0.0079 | 8.6 | 0.012 | 0.36 |
| 4.22 | 0.0079 | 15 | 0.02 | 0.80 |
| 4.22 | 0.0079 | 23.7 | 0.032 | 1.64 |
| 4.22 | 0.0079 | 27.8 | 0.037 | 2.11 |
| 4.22 | 0.0079 | 31 | 0.042 | 2.57 |
| 4.22 | 0.0079 | 33.9 | 0.046 | 3.02 |
| 1.85 | 0.0071 | 29.4 | 0.044 | 2.39 |
| 1.85 | 0.0071 | 33.9 | 0.051 | 3.08 |

In the second step of (38), the reciprocal value of $\tau(B)$ is replaced by $\xi$ defined in (8) and combined with (3) for $B = B_z$.

An examination of the ratio (4) can be reduced, for example, to only one diagonal component of the transversal magnetoresistance. In this case we obtain from (38)

$$\begin{aligned}&\frac{\Delta\rho_{xx}}{\rho_{xx}(0)} = \frac{\rho_{xx}^{\rm tot}(B) - \rho_{xx}(0)}{\rho_{xx}(0)} = \frac{\frac{1}{\tau(0)} + \frac{1}{\tau(B)} - \frac{1}{\tau(0)}}{\frac{1}{\tau(0)}} =\\ &\frac{\tau(0)}{\tau(B)} = \frac{B_z}{n_s e\xi c}\frac{n_s e^2\tau(0)}{m} = \frac{eB_z}{cm}\frac{\tau(0)}{\xi} = \frac{\Omega_0\tau(0)}{\xi}.\end{aligned} \quad (39)$$

This is a very simple result and its use is demonstrated

Table 3. Transversal magnetoresistance of one IIIa group metal (Al) calculated from (39) compared with the experimental data [19]. The meaning of the empirical correcting factor $r$ is the same as in the aforementioned tables.

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Al, single crystal: | | | | |
| 20 | 0.00184 | 5 | 0.122 | 0.77 |
| 20 | 0.00184 | 10 | 0.244 | 1.22 |
| 20 | 0.00184 | 20 | 0.487 | 1.81 |
| 20 | 0.00184 | 30 | 0.731 | 1.96 |
| 20 | 0.00184 | 34 | 0.828 | 2.06 |
| 14 | 0.00141 | 5 | 0.159 | 0.93 |
| 14 | 0.00141 | 10 | 0.318 | 1.48 |
| 14 | 0.00141 | 15 | 0.477 | 1.74 |
| 14 | 0.00141 | 20 | 0.636 | 1.91 |
| Al, polycrystal: | | | | |
| 77.2 | 0.161 | 5.2 | 0.0014 | 0.0008 |
| 77.2 | 0.161 | 10.3 | 0.0029 | 0.0025 |
| 77.2 | 0.161 | 14.6 | 0.0041 | 0.0048 |
| 77.2 | 0.161 | 20.1 | 0.0056 | 0.0087 |
| 77.2 | 0.161 | 23.5 | 0.0065 | 0.011 |
| 20.4 | 0.0675 | 4.1 | 0.0027 | 0.003 |
| 20.4 | 0.0675 | 10.3 | 0.0068 | 0.017 |
| 20.4 | 0.0675 | 14.6 | 0.0097 | 0.031 |
| 20.4 | 0.0675 | 20.1 | 0.0133 | 0.051 |
| 20.4 | 0.0675 | 23.5 | 0.0156 | 0.064 |

below in comparing the experimental data [19] with those calculated from (39); see Table 1 – 9. Formally, a formula similar to (39) is found in a study of a two-dimensional electron gas scattered by a random ensemble of antidots supplemented by a smooth random potential. In this case the magnetoresistance is a ratio of the transport time for the scattering of the antidot array and the transport scattering time by the long-range disorder [20].

The ratios (39) are usually considered at some temperature $T$ that influences $\tau(0)$. A factor of $r$, depending on $T$, couples $\tau(0) = \tau(B = 0, T)$ with $\tau(B = 0, T = 273\text{K})$, which is the relaxation time at $T = 273$K listed in Table 10 [21, 22]. We have for $T < 273$K [19]

$$\frac{1}{r}\tau(B = 0; T = 273\text{K}) = \tau(B = 0; T). \tag{40}$$

Because we regularly have $r < 1$ (see [19], exception are some cases of $r = 1$), the time $\tau(0)$ increases with a decrease of $T$. In fact, two kinds of $r$, one referring to zero degrees Celsius and the other referring to room temperature ($\sim$ 17 degrees Celsius), are used in [19]. In our treatment, we consider both kinds of $r$ on equal footing.

An application of (39) is facilitated if we note that the field of 1 Gauss (1 G) causes the cyclotron frequency

Table 4. Theoretical transversal magnetoresistance [see (39)] of one Va group metal (Nb) and two VIIIa group metals (Pd, Pt) compared with the experimental data [19]. The meaning of the correcting factor $r$ is the same as in the aforementioned tables.

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Nb, polycrystal: | | | | |
| 20.4 | 0.0682 | 40 | 0.014 | 0.0014 |
| Pd, polycrystal: | | | | |
| 78 | 0.17 | 100 | 0.030 | 0.02 |
| 78 | 0.17 | 200 | 0.061 | 0.06 |
| 78 | 0.17 | 300 | 0.091 | 0.10 |
| Pt, polycrystal: | | | | |
| 78 | 0.185 | 100 | 0.027 | 0.008 |
| 78 | 0.185 | 200 | 0.055 | 0.037 |
| 78 | 0.185 | 300 | 0.082 | 0.072 |
| 20.4 | 0.0067 | 8 | 0.06 | 0.04 |
| 20.4 | 0.0067 | 19.5 | 0.15 | 0.15 |
| 20.4 | 0.0067 | 26.4 | 0.20 | 0.23 |
| 20.4 | 0.0067 | 30.7 | 0.23 | 0.28 |
| 20.4 | 0.0067 | 35.8 | 0.27 | 0.35 |
| 20.4 | 0.0067 | 40.1 | 0.30 | 0.42 |
| Pt, polycrystal: | | | | |
| 14 | 0.0034 | 8 | 0.12 | 0.12 |
| 14 | 0.0034 | 19.5 | 0.29 | 0.40 |
| 14 | 0.0034 | 26.4 | 0.39 | 0.59 |
| 14 | 0.0034 | 30.7 | 0.46 | 0.72 |
| 14 | 0.0034 | 35.8 | 0.53 | 0.89 |
| 14 | 0.0034 | 40.1 | 0.60 | 1.07 |
| 4.22 | 0.0022 | 8 | 0.18 | 0.23 |
| 4.22 | 0.0022 | 19.5 | 0.45 | 0.75 |
| 4.22 | 0.0022 | 26.4 | 0.61 | 1.1 |
| 4.22 | 0.0022 | 30.7 | 0.70 | 1.3 |
| 4.22 | 0.0022 | 35.8 | 0.82 | 1.7 |
| 4.22 | 0.0022 | 40.1 | 0.92 | 2.0 |

$$\Omega_0^{1\text{G}} \cong 1.76 \times 10^7 \text{s}^{-1}, \tag{40a}$$

in which the number is expressed in radians. In the examined metals, we assumed that the magnetic field strength of 1 Oersted causes the induction strength almost equal to 1 Gauss [23]. Moreover, no corrections for the effective electron mass are considered in this paper.

Two values of $\xi$, equal to an elementary $\xi$ obtained in (8) and a quantum-mechanical $\xi$ of (34), respectively, can be applied. Computational practice – especially for alkali metals – shows, however, that generally $\xi$ of (8) fits the experimental data much better, and only the theoretical results based on $\xi$ of (8) are listed in the tables. The results corresponding to $\xi$ of (34) can be obtained from the tables by multiplying the theoretical data by a factor of

Table 5. Theoretical transversal magnetoresistance of the Ib group (noble) metals [see (39)] compared with experimental data [19]. The meaning of the empirical correcting factor $r$ is the same as in the aforementioned tables.

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Cu, polycrystal: | | | | |
| 78 | 0.141 | 100 | 0.11 | 0.1 |
| 78 | 0.141 | 200 | 0.21 | 0.3 |
| 78 | 0.141 | 300 | 0.32 | 0.5 |
| 77.2 | 0.148 | 23.6 | 0.024 | 0.01 |
| 63.8 | 0.094 | 23.6 | 0.038 | 0.02 |
| 20.4 | 0.02 | 23.6 | 0.178 | 0.19 |
| 14.2 | 0.02 | 23.6 | 0.178 | 0.19 |
| Cu, polycrystal: | | | | |
| 4.2 | 0.0089 | 30 | 0.51 | 0.5 |
| 4.2 | 0.0089 | 60 | 1.02 | 1.1 |
| 4.2 | 0.0089 | 90 | 1.53 | 2.0 |
| 4.2 | 0.0089 | 120 | 2.04 | 3.2 |
| 4.2 | 0.0089 | 150 | 2.55 | 4.6 |
| Ag, polycrystal: | | | | |
| 78 | 0.181 | 100 | 0.12 | 0.09 |
| 78 | 0.181 | 200 | 0.25 | 0.23 |
| 78 | 0.181 | 300 | 0.37 | 0.37 |
| Ag, rough crystal state: | | | | |
| 20.4 | 0.00293 | 4.58 | 0.35 | 0.51 |
| 20.4 | 0.00293 | 8.81 | 0.67 | 1.01 |
| 20.4 | 0.00293 | 10.85 | 0.83 | 1.33 |
| Au, polycrystal: | | | | |
| 79 | 0.219 | 18.5 | 0.014 | 0.005 |
| 79 | 0.219 | 25.5 | 0.02 | 0.010 |
| 79 | 0.219 | 31.5 | 0.024 | 0.012 |
| 79 | 0.219 | 35.8 | 0.027 | 0.018 |
| 79 | 0.219 | 40.1 | 0.031 | 0.018 |
| 20.4 | 0.0071 | 7.84 | 0.19 | 0.25 |
| 20.4 | 0.0071 | 15.3 | 0.36 | 0.59 |
| 20.4 | 0.0071 | 26 | 0.62 | 1.08 |
| 20.4 | 0.0071 | 33.9 | 0.80 | 1.43 |
| 20.4 | 0.0071 | 39.8 | 0.94 | 1.71 |
| Au, polycrystal: | | | | |
| 14 | 0.0023 | 7.84 | 0.57 | 0.9 |
| 14 | 0.0023 | 15.3 | 1.1 | 1.9 |
| 14 | 0.0023 | 26 | 1.9 | 3.2 |
| 14 | 0.0023 | 33.9 | 2.5 | 4.2 |
| 14 | 0.0023 | 39.8 | 2.9 | 5.0 |
| 4.22 | 0.00085 | 8 | 1.6 | 2.2 |
| 4.22 | 0.00085 | 15.8 | 3.1 | 4.5 |
| 4.22 | 0.00085 | 26.5 | 5.2 | 7.6 |
| 4.22 | 0.00085 | 34.2 | 6.8 | 9.7 |
| 4.22 | 0.00085 | 40.1 | 7.9 | 11.2 |

Table 6. Theoretical transversal magnetoresistance of the IIb group metals [see (39)] compared with experimental data [19]. The meaning of the empirical correcting factor $r$ is the same as in the aforementioned tables.

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Zn, polycrystal: | | | | |
| 291 | 1 | 300 | 0.008 | 0.06 |
| 195 | 0.68 | 300 | 0.012 | 0.12 |
| 78 | 0.19 | 100 | 0.014 | 0.18 |
| 78 | 0.19 | 200 | 0.029 | 0.57 |
| 78 | 0.19 | 300 | 0.043 | 0.93 |
| 77.2 | 0.202 | 23.6 | 0.003 | 0.02 |
| 20.4 | 0.0125 | 5.2 | 0.011 | 0.21 |
| 20.4 | 0.0125 | 10.3 | 0.023 | 0.49 |
| 20.4 | 0.0125 | 14.6 | 0.032 | 0.75 |
| 20.4 | 0.0125 | 20.1 | 0.044 | 1.12 |
| 20.4 | 0.0125 | 23.6 | 0.052 | 1.38 |
| 14.2 | 0.007 | 5.2 | 0.02 | 0.4 |
| 14.2 | 0.007 | 10.3 | 0.04 | 1.0 |
| 14.2 | 0.007 | 14.6 | 0.057 | 1.6 |
| 14.2 | 0.007 | 20.1 | 0.079 | 2.5 |
| 14.2 | 0.007 | 23.6 | 0.093 | 3.1 |
| Cd, polycrystal: | | | | |
| 291 | 1 | 300 | 0.009 | 0.08 |
| 195 | 0.68 | 300 | 0.014 | 0.19 |
| 78 | 0.22 | 100 | 0.014 | 0.24 |
| 78 | 0.22 | 200 | 0.029 | 0.58 |
| 78 | 0.22 | 300 | 0.043 | 0.93 |
| 77.2 | 0.2513 | 2.1 | 0.0003 | 0.0004 |
| 77.2 | 0.2513 | 4.1 | 0.0005 | 0.002 |
| 77.2 | 0.2513 | 10.3 | 0.0013 | 0.010 |
| 77.2 | 0.2513 | 14.6 | 0.0018 | 0.018 |
| 77.2 | 0.2513 | 23.6 | 0.0029 | 0.038 |
| Cd, polycrystal: | | | | |
| 20.4 | 0.023 | 2.1 | 0.0029 | 0.042 |
| 20.4 | 0.023 | 4.1 | 0.0056 | 0.12 |
| 20.4 | 0.023 | 10.3 | 0.014 | 0.43 |
| 20.4 | 0.023 | 14.6 | 0.020 | 0.66 |
| 20.4 | 0.023 | 23.6 | 0.032 | 1.16 |
| 14.2 | 0.0081 | 2.1 | 0.008 | 0.2 |
| 14.2 | 0.0081 | 4.1 | 0.016 | 0.5 |
| 14.2 | 0.0081 | 10.3 | 0.04 | 1.5 |
| 14.2 | 0.0081 | 14.6 | 0.057 | 2.2 |
| 14.2 | 0.0081 | 23.6 | 0.091 | 4.1 |
| Cd, single crystal: | | | | |
| 14 | 0.00645 | 16.3 | 0.08 | 1.8 |
| 14 | 0.00645 | 24.3 | 0.12 | 3.6 |
| 14 | 0.00645 | 29.4 | 0.14 | 5.0 |
| 14 | 0.00645 | 32.2 | 0.16 | 5.9 |
| 14 | 0.00645 | 35.1 | 0.17 | 6.9 |

$$\xi^{\text{elementary}}/\xi^{\text{quantum}} = 2\pi \tag{41}$$

which is the ratio of $\xi$ calculated in (8) and (34).

## 6. Discussion

A general result obtained in the present calculations is a systematic linear increase of the transversal magnetoresistance (4) with the strength $B$ of the magnetic field. This behaviour, dictated theoretically by the formula (39), is roughly confirmed by the experimental data listed in Tables 1 – 9.

A very accurate linear increase of magnetoresistance with an increase of the field $B$ has been obtained experimentally for various samples of metal-

Table 6 (continued).

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Cd, single crystal: | | | | |
| 20.4 | 0.0193 | 5.7 | 0.009 | 0.38 |
| 20.4 | 0.0193 | 11.2 | 0.018 | 0.85 |
| 20.4 | 0.0193 | 18.4 | 0.03 | 1.5 |
| 20.4 | 0.0193 | 24.3 | 0.04 | 2.0 |
| 20.4 | 0.0193 | 30.4 | 0.05 | 2.5 |
| 20.4 | 0.0193 | 36.6 | 0.06 | 2.9 |
| 14 | 0.0063 | 5.7 | 0.028 | 0.92 |
| 14 | 0.0063 | 11.2 | 0.056 | 2.01 |
| 14 | 0.0063 | 18.4 | 0.092 | 3.7 |
| 14 | 0.0063 | 24.3 | 0.12 | 5.3 |
| 14 | 0.0063 | 30.4 | 0.15 | 7.5 |
| 14 | 0.0063 | 36.6 | 0.18 | 9.4 |
| 4.22 | 0.00051 | 5.7 | 0.35 | 42 |
| 4.22 | 0.00051 | 11.2 | 0.69 | 151 |
| 4.22 | 0.00051 | 17.7 | 1.09 | 404 |
| 4.22 | 0.00051 | 24.6 | 1.51 | 677 |
| 4.22 | 0.00051 | 30 | 1.85 | 1020 |
| 4.22 | 0.00051 | 36.2 | 2.23 | 1425 |
| Cd, single crystal: | | | | |
| 14 | 0.00645 | 16.3 | 0.08 | 1.8 |
| 14 | 0.00645 | 24.3 | 0.12 | 3.6 |
| 14 | 0.00645 | 29.4 | 0.14 | 5.0 |
| 14 | 0.00645 | 32.2 | 0.16 | 5.9 |
| 14 | 0.00645 | 35.1 | 0.17 | 6.9 |

Table 7. Theoretical transversal magnetoresistance of the IIIb group metals [see (39)] compared with experimental data [19]. The meaning of the empirical correcting factor $r$ is the same as in the aformentioned tables.

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Ga, polycrystal: | | | | |
| 195 | 0.65 | 100 | 0.0015 | 0.033 |
| 195 | 0.65 | 200 | 0.0029 | 0.12 |
| 195 | 0.65 | 300 | 0.004 | 0.17 |
| 80 | 0.21 | 100 | 0.005 | 0.28 |
| 80 | 0.21 | 200 | 0.009 | 0.58 |
| 80 | 0.21 | 300 | 0.014 | 0.87 |
| Ga, polycrystal: | | | | |
| 165 | 0.63 | 300 | 0.01 | 0.03 |
| 80 | 0.22 | 100 | 0.01 | 0.01 |
| 80 | 0.22 | 200 | 0.019 | 0.06 |
| 80 | 0.22 | 300 | 0.029 | 0.14 |
| 78 | 0.2171 | 10 | 0.001 | 0.0006 |
| 78 | 0.2171 | 15 | 0.0015 | 0.001 |
| 78 | 0.2171 | 20 | 0.002 | 0.002 |
| 78 | 0.2171 | 25 | 0.0025 | 0.003 |
| 78 | 0.2171 | 30 | 0.0029 | 0.004 |
| 78 | 0.2171 | 35 | 0.0034 | 0.006 |
| 78 | 0.2171 | 40 | 0.0039 | 0.008 |
| 20.4 | 0.023 | 10 | 0.009 | 0.02 |
| 20.4 | 0.023 | 15 | 0.014 | 0.04 |
| 20.4 | 0.023 | 20 | 0.019 | 0.06 |
| 20.4 | 0.023 | 25 | 0.023 | 0.08 |
| 20.4 | 0.023 | 30 | 0.028 | 0.11 |
| 20.4 | 0.023 | 35 | 0.032 | 0.14 |
| 20.4 | 0.023 | 40 | 0.037 | 0.17 |
| In, polycrystal: | | | | |
| 14 | 0.00855 | 10 | 0.025 | 0.09 |
| 14 | 0.00855 | 15 | 0.037 | 0.16 |
| 14 | 0.00855 | 20 | 0.050 | 0.23 |
| 14 | 0.00855 | 25 | 0.062 | 0.30 |
| 14 | 0.00855 | 30 | 0.075 | 0.38 |
| 14 | 0.00855 | 35 | 0.087 | 0.45 |
| 14 | 0.00855 | 40 | 0.10 | 0.52 |
| 4.22 | 0.00119 | 10 | 0.18 | 0.84 |
| 4.22 | 0.00119 | 15 | 0.27 | 1.02 |
| 4.22 | 0.00119 | 20 | 0.36 | 1.12 |
| 4.22 | 0.00119 | 25 | 0.45 | 1.19 |
| 4.22 | 0.00119 | 30 | 0.54 | 1.25 |
| 4.22 | 0.00119 | 35 | 0.63 | 1.28 |
| Tl, polycrystal: | | | | |
| 80 | 0.23 | 100 | 0.0054 | 0.025 |
| 80 | 0.23 | 200 | 0.011 | 0.087 |
| 80 | 0.23 | 300 | 0.016 | 0.159 |

Table 8. Theoretical transversal magnetoresistance of the IVb group metals [see (39)] compared with experimental data [19]. The meaning of the empirical correcting factor $r$ is the same as in the aformentioned tables.

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Sn, polycrystal: | | | | |
| 291 | 1 | 300 | 0.0039 | 0.02 |
| 80 | 0.22 | 100 | 0.0059 | 0.043 |
| 80 | 0.22 | 200 | 0.012 | 0.13 |
| 80 | 0.22 | 300 | 0.018 | 0.23 |
| Pb, polycrystal: | | | | |
| 291 | 1 | 300 | 0.0024 | 0.01 |
| 80 | 0.25 | 300 | 0.0094 | 0.05 |
| 20.4 | 0.02965 | 8 | 0.0021 | 0.0026 |
| 20.4 | 0.02965 | 15.8 | 0.0042 | 0.0078 |
| 20.4 | 0.02965 | 24.4 | 0.0065 | 0.018 |
| 20.4 | 0.02965 | 31.9 | 0.0084 | 0.031 |
| 20.4 | 0.02965 | 39.8 | 0.011 | 0.047 |
| 14 | 0.01052 | 8 | 0.006 | 0.017 |
| 14 | 0.01052 | 15.8 | 0.012 | 0.060 |
| 14 | 0.01052 | 24.4 | 0.018 | 0.12 |
| 14 | 0.01052 | 31.9 | 0.024 | 0.19 |
| 14 | 0.01052 | 39.8 | 0.03 | 0.27 |
| 4.22 | 0.000175 | 8.0 | 0.36 | 12 |
| 4.22 | 0.000175 | 15.8 | 0.71 | 37 |
| 4.22 | 0.000175 | 24.4 | 1.1 | 74 |
| 4.22 | 0.000175 | 31.8 | 1.4 | 116 |
| 4.22 | 0.000175 | 40 | 1.8 | 174 |

lic potassium [24]. The observed $\Delta\rho$ changed linearly with $B$, especially for high $B$, with no tendency of saturation expected by the former theories [25]. This behaviour has been confirmed experimentally not only for polycrystalline but also for monocrystalline potassium samples [26].

Table 9. Theoretical transversal magnetoresistance of the IVb group metals [see (39)] compared with experimental data [19]. The meaning of the empirical correcting factor $r$ is the same as in the aforementioned tables.

| temperature (in K) | correcting factor $r$ | field intensity $B$ (in kG) | theoretical $\Delta\rho/\rho(0)$ | experimental $\Delta\rho/\rho(0)$ |
|---|---|---|---|---|
| Sb, polycrystal: | | | | |
| 291 | 1 | 100 | 0.0003 | 0.8 |
| 291 | 1 | 200 | 0.0006 | 2.2 |
| 291 | 1 | 300 | 0.0009 | 3.5 |
| 195 | 0.67 | 100 | 0.0005 | 1.7 |
| 195 | 0.67 | 200 | 0.0009 | 4.8 |
| 195 | 0.67 | 300 | 0.0014 | 8.0 |
| Sb, polycrystal: | | | | |
| 80 | 0.3 | 100 | 0.001 | 7 |
| 80 | 0.3 | 200 | 0.0021 | 23 |
| 80 | 0.3 | 300 | 0.0031 | 40 |
| Sb, single crystal: | | | | |
| 79.2 | 0.1895 | 3.43 | 0.0001 | 0.15 |
| 79.2 | 0.1895 | 6.75 | 0.0001 | 0.40 |
| 79.2 | 0.1895 | 10.1 | 0.0002 | 0.74 |
| 79.2 | 0.1895 | 12.1 | 0.0002 | 1.01 |
| Sb, single crystal: | | | | |
| 78.1 | 0.1826 | 3.43 | 0.0001 | 0.15 |
| 78.1 | 0.1826 | 6.75 | 0.0001 | 0.43 |
| 78.1 | 0.1826 | 10.1 | 0.0002 | 0.81 |
| 78.1 | 0.1826 | 12.1 | 0.0002 | 1.11 |
| Sb, single crystal: | | | | |
| 77.8 | 0.1763 | 3.43 | 0.0001 | 0.11 |
| 77.8 | 0.1763 | 6.75 | 0.0001 | 0.31 |
| 77.8 | 0.1763 | 10.1 | 0.0002 | 0.60 |
| 77.8 | 0.1763 | 12.1 | 0.0002 | 0.81 |
| Bi, polycrystal: | | | | |
| 291 | 1 | 300 | 0.0004 | 37 |
| 195 | 0.67 | 300 | 0.0006 | 196 |
| 80 | 0.346 | 300 | 0.0011 | 1360 |
| Bi, single crystal: | | | | |
| 14.15 | 0.024 | 4.0 | 0.0002 | 260 |
| 14.15 | 0.024 | 10.1 | 0.0005 | 1506 |
| 14.15 | 0.024 | 15.1 | 0.0008 | 3544 |
| 14.15 | 0.024 | 20.3 | 0.0011 | 4655 |
| 14.15 | 0.024 | 25.0 | 0.0013 | 5483 |
| 14.15 | 0.024 | 30.8 | 0.0017 | 11954 |

In Table 11, we calculated the average value (the arithmetical mean) of the ratio

$$s = \frac{(\Delta\rho/\rho(0))^{\mathrm{theor}}}{(\Delta\rho/\rho(0))^{\mathrm{exp}}} \tag{42}$$

for the metals examined in Tables 1 – 9. Only metals for which several measurements were taken into account are considered in these kind of statistics. The ratios presented in Table 11 give a sense of the accuracy of the theory developed in the present paper.

Table 10. Relaxation times $\tau(0)$ (in $10^{-14}$ s) from [21] and [22] applied in Tables 1 – 9. These $\tau(0)$ refer to the temperature 273 K and are corrected for the measurements temperatures by dividing $\tau(0)$ by the empirical factor $r$ given in Tables 1 – 9.

| | | | | | |
|---|---|---|---|---|---|
| Li | 0.88 | Be | 0.51 | Pd | 0.92 |
| Na | 3.2 | Mg | 1.1 | Pt | 0.9 |
| K | 4.1 | Ba | 0.19 | Cu | 2.7 |
| Rb | 2.8 | Al | 0.8 | Ag | 4.0 |
| Cs | 2.1 | Nb | 0.42 | Au | 3.0 |
| Zn | 0.49 | Ga | 0.17 | Sn | 0.23 |
| Cd | 0.56 | In | 0.38 | Pb | 0.14 |
| | | Tl | 0.22 | | |
| Sb | 0.055 | Bi | 0.023 | | |

Table 11. A control of the accuracy of the theory: the average value of the ratio $s$ of the formula (42) calculated for the metal cases presented in Tables 1 – 9.

| | | | |
|---|---|---|---|
| Li | Na | K | |
| 1.0 | 5.2 | 4.1 | |
| Be | Mg | Ba | |
| 0.017 | 0.10 | 0.03 | |
| Pd | Pt | | |
| 1.15 | 0.91 | | |
| Cu | Ag | Au | |
| 1.04 | 0.91 | 1.02 | |
| Zn | Cd | | |
| 0.06 | 0.09 | | |
| Al | Ga | In | Tl |
| 0.50 | 0.02 | 0.45 | 0.15 |
| Sn | Pb | | |
| 0.12 | 0.22 | | |
| Sb | Bi | | |
| $3.0\times10^{-4}$ | $1.7\times10^{-6}$ | | |

Metals like Li, Pd, Pt, Cu, Ag, Au, and Al have their average ratio $s$ from (42) within, or at the limit, of the interval

$$0.5 < s^{\mathrm{av}} < 1.5 \tag{43}$$

which indicates good agreement between experiment and theory. Less agreement is found for the metals whose average $s$ is either within

$$0.1 < s^{\mathrm{av}} < 0.5 \tag{44a}$$

(Mg, In, Tl, Sn, Pb) or

$$0.01 < s^{\mathrm{av}} < 0.1 \tag{44b}$$

(Be, Ba, Zn, Ca, Ga). Special cases of $s^{\mathrm{av}}$ concern Sb and Bi. The $s^{\mathrm{av}}$ in these metals differ by several orders from the lower limit of (44a), indicating a predominantly open character of the electron orbits. The largest theoretical magnetoresistance (4) presented in the tables is 7.9 for Au ($T = 4.22$ K and $B = 40.1$ kG). Its experimental counterpart amounts to 11.2. In calculating the theoretical data for Au only polycrystalline samples were taken into account: the monocrystals of Au seem to make $s^{\mathrm{av}}$ very much outside the interval (43) and this can be connected with special properties (giant fluctuations) of the magnetoresistance observed with the direction changes of the magnetic field [27], which are not considered in the present paper.

From the individual metal cases examined in Tables 1 – 9 Rb gives the ratio $s$ in (42) far above the upper limit of the interval in (43). The same property characterizes $s$ of Nb, but Cs has its $s$ rather close to the upper limit of (43).

## 7. Summary

A method is presented in which the transversal magnetoresistance in metals is calculated and is compared with experimental data. Numerous metal cases, corresponding to various magnetic field strengths and measurement temperatures, were examined. To the best of our knowledge, no comparison of a similar extent using magnetoresistance data has been previously undertaken.

The theory is based on an assumption that two kinds of relaxation time, one due to a stationary electric field and another one corresponding to the action of the magnetic field alone, should be taken into account. Simultaneously, the band structure of a metal is simplified to a single band of states for which no particular effective mass is incorporated into the formalism. In addition, the problem of preparing the metal samples, which can be an important factor in searching for reasons for the disagreement between the experiment (see e. g. [24]) and theory, has been neglected.

Agreement between the results of the present method and the experimental data depends strongly on the kind of metal examined. The best agreement is found for metals like Li, Pd, Pt, and noble metals (Cu, Ag, Au). Fair agreement is attained for Al and In.

Metals like Sb and Bi are found to be beyond of the area of approach provided by the present theory. A reason for that is a predominantly open character of the electron orbits on the Fermi surfaces; see e. g. [13].

Preliminary calculations leading to the properties of the relaxation time in the presence of the magnetic field similar to those applied in the present paper have been done in [28 – 31].

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